(b) Find a matrix P which transforms the matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ into a diagonal form and hence find A^4. **4+3½**

Unit III

5. (a) A square piece of tin of side 18 cm. is to be made into a box without top by cutting a square from each corner, and folding up the flaps to form a box. What should be side of the square to be cut off so that volume of the box is maximum ? Also find this maximum volume. **5+2½**

(b) Expand $\sin x$ in power of $\left(x - \frac{\pi}{2}\right)$ and hence find the value of $\sin 9i$ correct of four decimal places. **4+3½**

6. (a) Find equation of the evolute of the curve $x^2 = 4ay$. **7½**

No. of Printed Pages : 06 **Roll No. ....**

18A2

B.Tech. EXAMINATION, 2022

(First Semester)

(C-Scheme) (Main & Re-appear)

(CSE)

MATH101C

MATHEMATICS-I

Time : 3 Hours] [*Maximum Marks :* 75

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **9** is compulsory. All questions carry equal marks.

Unit I

1. (a) If $A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ p & q & r\end{bmatrix}$ and I is the identity matrix of order 3, show that : **7½**

$$A^3 = pI + qA + rA^2$$

(b) Show that : **7½**

$$\begin{vmatrix}a+b+2c & a & b\\ c & b+c+2a & b\\ c & a & c+a+2b\end{vmatrix}=2(a+b+c)^3$$

2. (a) Find A^{-1}, if $A=\begin{bmatrix}1 & 2 & 5\\ 1 & -1 & -1\\ 2 & 3 & -1\end{bmatrix}$. Hence solve the system : **4+3½**

$$x + 2y + 5z = 10$$
$$x - y - z = -2$$
$$2z + 3y - z = -11$$

(b) For what value of λ, the following system is consistent : $\lambda x+y+z=6; x-y+2z=5; 3x+y+z=8$? Also find its solution. **4+3½**

Unit II

3. (a) Find the eigen value and the eigen vectors of the matrix : **4+3½**

$$\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}$$

(b) Express the matrix $A=\begin{bmatrix}4 & 2 & -1\\ 3 & 5 & 7\\ 1 & -2 & 1\end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix. **7½**

4. (a) Define orthogonal transformation. Verify that $\begin{bmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ \sin\theta & 0 & \cos\theta\end{bmatrix}$ is an orthogonal matrix. **2+5½**

(c) For what values of x the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & x^2-9 & 5 \\ 2 & -5 & 0 \end{bmatrix}$ is skew-symmetric.

(d) Evaluate :

$$\int_0^{\pi} \sin^3 x \, dx$$

(e) Evaluate by using Gamma functions :

$$\int_0^{\infty} e^{-x^2} x^5 dx$$

(f) Find the volume of the solid generated by revolving the circle $x^2 + y^2 = a^2$ about x-axis. **2½×6=15**

(b) Find all asymptotes of the curve :

$x^3 + 3x^2y - xy^2 - 3y^3 + x^2 - 2xy + 3y^2 + 4x + 7 = 0.$ 7½

Unit IV

7. (a) Show that any field forms a vector space over itself. **7½**

(b) A non-empty subset W of a vector space V(F) is a sub-sapce of V iff $au + bv \in W$ for $a, b \in F$ and $u, v \in V$. **7½**

8. State and prove Rank-Nullity theorem. **15**

(Compulsory Question)

9. (a) Let R be a field of real numbers, then show that $W = \{(x, x, x) : x \in R\}$ is a subspace of $V_3(R)$.

(b) Find rank of the matrix :

$$\begin{bmatrix} 2 & 1 & -2 \\ 1 & -2 & 1 \\ 5 & -5 & 1 \end{bmatrix}$$